Theory and Applications for Econometrics

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Test on Course Prerequisites

1. Suppose that a discrete random variable X has probability mass function $p(x)$ given by the following table:

x	0	1	2
$p(x)$	2/6	1/6	3/6

   (a) find the cumulative mass function of X; (b) find the expected value (mean) $\mathsf{E}[X]$ and the variance $\mathsf{Var}[X]$ of X.

2. Suppose that a discrete random variable X, taking only the values 0 and 1, has probability mass function $p(x)$ given as $p(x;\theta) \stackrel{\text{def}}{=} \theta^x(1-\theta)^{1-x}$ with θ being an unknown parameter. (a) verify that $p(x)$ is indeed a proper probability mass function; (b) find the expected value (mean) $\mathsf{E}[X]$ and the variance $\mathsf{Var}[X]$ of X.

3. Suppose that a continuous random variable X follows the uniform distribution in the interval $[a, b]$. That is, it has probability density function given by $f(x) \stackrel{\text{def}}{=} 1/(b-a)$. (a) find the cumulative density function of X; (b) find the quantile function of X; (c) find the expected value (mean) $\mathsf{E}[X]$ and the variance $\mathsf{Var}[X]$ of X.

4. Consider two random variables X and Y. How do we define their covariance and how do we define their correlation?

5. You are given a random variable X and constant parameters α and β. If the expected value and variance of X are given by $\mathsf{E}[X] \stackrel{\text{def}}{=} \mu$ and $\mathsf{Var}[X] \stackrel{\text{def}}{=} \sigma^2$ what are the expected

value and variance of the new random variable $Y \stackrel{\text{def}}{=} \alpha + \beta X$? In addition, what are the expected value and variance of the new random variable $Z \stackrel{\text{def}}{=} (X - \mu)/\sigma$?

6. You are given a random sample of observations for the random variable X, say $\{x_i\}_{i=1}^n$ where n is the sample size. (a) how do you define the sample mean $\bar{x}_n$? (b) how do you define the sample variance s_n^2?

7. In the context of the previous question, show that the sample mean is the solution to the minimization problem $\bar{x}_n = \min_\theta n^{-1} \sum_{i=1}^n (x_i - \theta)^2$, where θ denotes a parameter.

8. In the context of question 7, suppose that you know that your sample values are drawn from a normal distribution with mean μ and variance σ^2, i.e. you have $x_i \sim N(\mu, \sigma^2)$. Using this additional information (a) can you find the expected value and variance of the sample mean, i.e. can you find $\mathsf{E}[\bar{x}_n]$ and $\mathsf{Var}[\bar{x}_n]$? How do you interpret your answers? (b) can you find the sampling distribution of $\bar{x}_n$?

9. In the context of the previous question, can you show how you can construct a confidence interval for the unknown mean μ based on the sample mean and assuming that σ^2 is known?

10. In the context of question 9, suppose now that you wish to test whether the true but unknown value of μ is zero, still assuming that σ^2 is known. You set up a hypothesis test for the null hypothesis $H_0 : \mu = 0$ against the alternative $H_1 : \mu \neq 0$. Show how you can test the null hypothesis and indicate the test statistic that you will use.

11. Suppose that you are given a random sample of observations for the random variables X and Y, say $\{x_i, y_i\}_{i=1}^n$ where n is the sample size. (a) how do you define the sample covariance and sample correlation? (b) what exactly does the sample correlation measures?

12. In the context of the above question, consider the simple regression model $y_i = \alpha + \beta x_i + u_i$ where (α, β) are the unknown parameters of the regression and u_i is the regression error term, assumed to follow a normal distribution $u_i \sim N(0, \sigma^2)$. Discuss how you will estimate the parameters of the regression and comment on anything else that you are aware of about this model (e.g. goodness of fit, test of hypotheses etc.)